Detecting Multiple Bubbles and Exuberance in Financial Data: An Extensive Empirical Examination over Four Major Foreign Indexes.

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Abstract

History is replete with incidents of financial crisis, which ex-post become a wakeup call for policy makers and the people. But there were no tests which could identify and date financial bubbles in real time, till now. Phillips, Shi and Yu [2015] provides the first and only model to recursively examine for multiple bubbles. Their "flexible window" methodology provides consistent results and has successfully identified the well-known historical episodes of exuberance and collapse. This accuracy provides very useful "warning alerts" to central bankers, fiscal regulators and policy makers to pre-emptively act and possibly eliminate an impending implosion.

We extensively examine for the presence and recurrence of multiple bubbles, over four major financial indexes. We find evidence of bubbles and explosive sub-periods over the long-term data for all of the indices, including deciphering the technology bubbles of the 1990s and early 2000s, and the financial crises of 2008.

Keywords: Financial bubbles, Financial crisis, Multiple bubbles, SADF, GSADF.
JEL Codes: F65, G10.

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INTRODUCTION

History is replete with incidents of financial crisis, which ex-post become a wakeup call for policy makers and the people. Again, and again it was stated by experts that the present crisis was preceded by "asset market bubbles" and / or "excessive credit expansion." But the fact of the matter remains that we do not have good quantitative markers which can ex-ante indicate the genesis of a momentum being built in the asset / credit markets which may lead to a catastrophe down the line. Thus, we had to accept that there was no practical way to identify the "red flags" of a crisis. Thus, the task at hand is to try to decipher possible quantitative markers from the data, that a speculative bubble is probably taking shape.

In the economics literature we have multiple tests to detect ex-post the crisis, and then explain it. (1) But there was no test to ex-ante identify the origination of a bubble which is in the making, i.e., there were no econometric detectors of a future market crisis. Phillips, Wu and Yu [PWY henceforth, 2011] presented a recursive method to detect exuberance in asset prices during an inflationary phase. The advantage here being that the early detection (ex-ante acknowledgement) can help banks / regulators / policy makers to address the problem in its nascent state. PWY was very effective in the early detection of bubbles, provided there was a single bubble in the data sample. They proved the effectiveness of the test using NASDAQ PWY [2011] and the US housing bubble in Phillips and Yu [PY henceforth, 2011].

But then came the question of "economic reality" which showed that there usually were multiple recurring financial crises, over long periods. Ahmed [2009] gave us evidence of 60 different financial crises, in the 17th century alone. A test to clearly identify periodic collapsing and recovering economic data was simply not there. Thus, the next step in the evolution of these detection tests was to create the one that could decipher multiple bubbles in the same sample period. This recursive identification is extremely complex, compared to identifying a single bubble. The main problem is computationally handling the non-linear structure of multiple breaks / bubbles in the data. With the presence of multiple break points in the data, the discriminatory power of the detectors goes down dramatically, and hence the upswings and downswings are not separable in the same data stream.

The challenge here was not only to come up with a statistical metric which can detect multiple factual fractures in the non-linear data stream, but at the same time, also be powerful and effective enough to distinguish between a false negative detection (to avoid unnecessary policies) and a true positive detection tolerance (so as to ensure good and early effective policy application.)

This is where the Phillips, Shi and Yu [PSY henceforth, 2014] research comes into effect. This paper offers the first powerful and credible "quantitative metric" to detect exuberance in financial data, right where it is originating. Once detected, the counteractive policies can be promulgated and implemented. Looking at long term S&P 500 data from 1871 - 2010 (about 140 years), the authors propose a recursive algorithm, which can diagnose and identify ex-ante the signs of "turbulence within the force" if you will. This procedure helps us pinpoint the start of the problem and can thus help us monitor the markets. Since we know that history has proven that it has a bad habit of repeating itself, this early warning diagnostic tool will come in handy, in helping make / alter policies to avert the impending crisis. The best part of this test is that it can be implemented on current data in real time to detect the "fault lines."

PSY [2014] presents a recursive econometric technique to detect / test / date financial bubbles in the same sample data and separate them when multiple bubbles are present. Here the authors extend on their [PWY, 2011] methodology, which is based on a sequence of forward recursive right tailed ADF unit root tests, using the Sup ADF (designated SADF) measure. This process allows for a dating strategy to identify the origination and termination dates of a specific bubble. This is achieved by using "backward regression techniques." In case of a single bubble, the PWY test is consistent, (as shown in Phillips and Yu 2009.) This detection algorithm is better able to date the ups and downs of financial data, as opposed to the CHOW tests, CUSUM tests etc. as evidenced by Homm and Breitung [2012].

But what if there are multiple bubbles, originating and decaying in sequence over time. PWY is not proven to be consistent in such cases. It cannot be confidently used in examining long term market data where exuberance and collapse are evident ex-post. Here PSY [2014] present an extension of the SADF tests, in form of a generalized SADF called the GSADF method. It includes a recursive backward regression technique, to time identify the origin and collapse of bubbles. It is a right tailed ADF test but has a flexible window width to separate one bubble from the next, to the next sequentially, since their lengths are bound to be different. It's an ex-ante procedure to detect different start and end points of bubbles in real time data, i.e., identify and separate multiple bubble episodes over the same sample set. This test has been proven to consistently give good results, when multiple bubbles are present. Thus, it can credibly be applied to analyzing long term historical data. Along with the ex-ante dating algorithm and the GSADF test, the authors develop a modified PWY algorithm, which reinitializes the test sequentially, after the detection of each bubble. This sequential test works in deciphering multiple bubbles from explosion to collapse and separate them over time. It is applied to the S&P 500 stock market data from January 1871- December 2010. It has been able to identify all the historically documented bubble episodes, like the 1929 crash, 1954 boom, 1987 black Monday and the latest dot-com bubble.

In quite possibly a first, we use this powerful metric, to extensively examine for multiple bubbles over four major data indices, namely the FTSE 100, CAC, DAX and the NIKKEI. (2) Section 2 describes the reduced form model, the new rolling window recursive test and its limit theory. Section 3 elaborates the data stamping strategies to identify and separate multiple bubbles in the same sample period, and discusses the size, power and performance of the dating strategy tests. In section 4, we apply the PWY test, the sequential PWY test and the CUSUM test, and do an extensive examination for the presence of multiple bubbles in all four of the above-mentioned foreign indexes. Section 5 concludes.

SECTION 1. ROLLING WINDOW TEST FOR BUBBLES

It originates with the standard asset pricing model(3)

$$P_t = \sum_{i=0}^{\infty} \left(\frac{1}{1+r_f}\right) i \, E_t(D_{t+i} + U_{t+i}) + B_t \qquad (1)$$

where

P_t = after dividend price of an asset

D_t = payoff (dividend) from the asset

r_f = risk free interest rate

U_t = unobservable fundamentals

B_t = bubble component

Here $P_t^f = P_t - B_t$ (market fundamentals) and B_t satisfies the sub martingale property

$$E_t = (B_{t+1}) = (1 + r_f)B_t \tag{2}$$

This equation sets up the alternative scenarios for the presence / absence of bubbles in the data. For example: If there are no bubbles, the B_t =0, then the degree of non-stationarity [I(0) or I(1)] of asset prices is controlled by asset payoffs or dividends (D_t) and the unobservable economic / market fundamentals. A possible outcome would be like this: If D_t is an I(1) process, the U_t has to be either I(0) or I(1) and asset prices can at the most be a I(1) process. But based on eq. (2), if there are bubbles, then asset prices will be explosive. Thus, when the fundamentals are I(1) and D_t is first difference stationary, we can infer bubbles if asset prices show evidence of explosive behavior. Eq (1) is one way to include a bubble variable in the standard asset pricing model, but the jury is still out on this. [4, 5] The advantage of the reduced form model is that it pretty much encompasses all standard formulations as intrinsic bubbles [Froot and Obstfeld, 1991], herd behavior [Abreu and Brunnermeier, 2003] and also time varying discounting [Phillips and Yu, 2011.] Shi [2011] provides an excellent overview of this literature.

According to Phillips and Magdalinos [2007], explosive behavior in asset prices is a primary indicator of market exuberance, which can be identified in empirical tests using the "recursive testing procedure" like the right-side unit root test of PWY. This recursive procedure starts with a martingale null (with drift to capture long historical trends in asset data.) The model specification is:

$$y_t = dT^{-n} + \theta_{yt-1} + \epsilon_t \tag{3}$$

where €$_t$ is iid (0, σ^2), $\theta = 1$, and d is a constant, T is the sample size, and the parameter η controls the magnitude of the intercept and the drift, as $T \rightarrow \infty$. Solving eq. 3, gives us the deterministic trend, dt/T^n. The three possibilities here (in sequence) are that if n>0, the drift will be small compared to the linear trend, if n>1/2, the drift is small relative to the martingale and if n=1/2, the output behaves like a Brownian motion, which is evident in many financial time series data.

The emphasis is on the alternative hypothesis, because departures from market fundamentals are the markers of interest. But as with all types of model specifications, we know that they are sensitive to intercepts, trends and trend breaks etc. Eq. 3 is tested for exuberance using the rolling window ADF approach or the recursive approach. The basic logic is that if the rolling window regression starts from the r_1^{th} fraction and ends with the r_2^{th} fraction (from sample size T), then $r_2 = r_1+r_w$, where r_w is the size of the window. This model is:

$$\Delta y_t = \propto_{r1,r2} + \beta_{r1,r2}\, y_{t-1} + \Sigma_{i=1}^{k}\, \gamma_{r1,r2}^{i}\, \Delta_{yt-1} + \in_t \qquad (4)$$

where k is the lag length, and $\in_t$ is iid, with $(0, \sigma^2_{r1,r2})$. The basic form is reformulated to include the presence of "multiple bubbles" to separate the market switching time periods from explosion to contraction, and again explosion sequentially. They use the Sup ADF test called SADF. It is a recursive / repeated estimation procedure with window size r_w., where r_w goes from r_0 (smallest sample window fraction) to r_1 (largest sample window fraction), and sample end point $r_2 = r_w$, going from 0 to 1. The SADF statistic is: [6]

SADF (r_0) n= $\sup_{r2\in[r0,1]} ADF^{r2}_0$ (4a)

The ADF regression is run on eq. 4, recursively, but continuously on sub-samples of the data based on window width chosen according to r_0, r_1, r_2……r_w. The subsamples chosen here are more extensive than the SADF test. The difference here is that we allow the window width to change within the feasible range where $r_w = r_2 - r_1$.

The GSADF statistic is:

GSADF (r_0) = sup $r_2 \in [r_0, 1]$ $\{ADF^{r2}_{r1}\}$ (5)

$r_1 \in [0, r_2 - r_0]$

The limit distribution of the GSADF holds, but with the intercept and the assumption of a random walk structure, we have no drift or small drift. The GSADF's asymptotic distribution depends on the "smallest window width size r_0." It depends on the number of observations in the sample. If T is small, r_0 has to be made large enough to ensure the inclusion of an adequate number of observations. But, if T is large, r_0 should be set small, so as to be able to include different "explosive" burst in the data. Simulations in PSY (2014] show that as r_0 decreases, the critical values (CV's, henceforth) of the test statistic increases. GSADF statistic CV's are larger than the SADF statistic, which in turn is larger than the ADF statistic, and its concentration also increases, increasing confidence in the test outcomes. The backward SADF statistic is the sup value of the ADF sequence run over this interval, BSADF r_2 (r_0) = sup $_{r1 \in [0,\, r2-r0]}$ $\{ADF^{r2}_{r1}\}$.

Empirically we determine the ADFr_2 and the sup ADF within the feasible range of r_2 (from r_0 to r_1.) This procedure imposes the condition that the bubble marker is the existence of a critical value greater than L_T = Log (T). This separates the short and temporary market blips (which happen all the time in real life) from actual exuberance. Dating is done using the formula:

$$\hat{r_e} = inf_{r2 \in [r0,1]}\{r_2 : ADF_{r2} > cv^{\beta T}_{r2}\} \qquad (6)$$

and

$$\hat{r_f} = inf_{r2 \in [\hat{r_e} + \frac{\log(T)}{T,1}]}\{r_2 : ADF_{r2} < cv^{\beta T}_{r2\}}\} \qquad (7)$$

where $cv^{\beta T}_{r2}$ is the 100(1-$\boldsymbol{\beta}_T$) % critical value of the ADF statistic based on [T_{r2}] observations. Here $\boldsymbol{\beta}_T \rightarrow 0$, as $T \rightarrow \infty$.

SECTION 2. DATA STAMPING STRATEGIES

The idea is to identify bubbles in real time data and then look for the "markers" identifying those bubbles / episodes of market exuberance. The problem is that the standard ADF test can identify extreme observations, as r = $[T_r]$, but cannot separate between a bubble phase observation from one which is part of a natural growth trajectory. Market growth is not an indication of bubbles. Thus, ADF tests may result in finding "pseudo bubble detection." So, how to make this distinction is the major contribution of this PSY (2014] test. The authors run backward sup ADF or backward SADF tests, to improve the chances of deciphering a bubble from a growth trajectory. The recursive test means running SADF backwards on the sample, increasing the sample sequence using a fixed sample r_2, but varying the initial point from 0 to (r_2-r_0). This gives the SADF statistic: $\{ADF^{r2}{}_{r1}\} \in |0, r_2 \text{-}0|$. Bubbles are inferred from the backward SADF statistic or the BSADF $r_2(r_0)$. The origin of the bubbles, the date and timing is the first observation whose BSADF statistic exceeds the critical value of the BSADF. The bubble ending date / time frame is the first observation whose BSADF is below the BSADF critical value. The intermediary time frame is the duration of the bubble. The origination / termination dates are calculated thus:

$$\hat{r_e} = inf_{r2\in[r0,1]}\{r_2: BSADF_{r2}(r_0) > scv_{r2}^{\beta T} \tag{8}$$

$$\hat{r_f} = r_{2\in}[inf_{\hat{r_e}+\left[\frac{\partial \log(T)}{T,1}\right]}\{r_2: BSADF_{r2}(r_0) > scv_{r2}^{\beta T} \tag{9}$$

where $scv\boldsymbol{\beta}^{T}{}_{r2}$ is the $100(1\text{-}\boldsymbol{\beta}_T)\%$ critical value of the sup ADF statistic, based on $[T_{r2}]$ observations. $\boldsymbol{\beta}_T$ goes to zero, as the sample size approaches infinity. The distinction between the SADF and the GSADF (backward sup ADF) tests, both run over $r_2 \in[r_0,1]$ is given by the statistic, SADF (r_0) = $sup_{r2\in[r0,1]}$ $\{ADF_{r2}\}$ and GSADF (r_0) = $sup_{r2\in[r0,1]}$ $\{BSADF_{r2}(r_0)$. The authors [PSY, 2014] elaborate on the details and derivations of the limit theorems for bubble identification covering all cases, from normal asset price trajectories, i.e., no bubbles to identification of single and most importantly multiple bubbles. [7] The empirical process for detection of multiple bubbles involves more complex dating strategies. The model generation equation is:

$$X_t = X_{t-1}\{t \in N_0\} + \delta_t X_{t-1} 1\{t \in B_1 \cup B_2 + \left(\Sigma_{k=r1f+1}^{t} \in_k + X_{r1f}^*\right) 1\{t \in N_1\} +$$

$$\left(\Sigma_{t=r2f+1}^{t} \in_t + X_{r2f}^*\right) 1\{t \in N_2\} + \in_t 1\{j \in N_0 \cup B_1 \cup B_2\} \tag{10}$$

where $N_0 = [1+\tau_{1e})$, $B_1 = [\tau_{1e}, \tau_{1f}]$, $N_1 = [\tau_{1f}, \tau_{2e}]$, $B_1 = [\tau_{2e}, \tau_{2f}]$, and $N_2 = (r_{2f}, \tau]$. The observations $\tau_{1e} = [T_{r1e}]$ and $\tau_{1f} = [T_{r1f}]$ are the origination and termination dates of the first bubble. Similarly, $\tau_{2e} = [T_{r2e}]$ and $\tau_{2f} = [T_{r2f}]$ is the origination and termination dates of the second bubble, where τ is the last observation in the sample. Once the first bubble collapses, X_t resumes its normal martingale path till $[r_{2e}-1]$, where the second bubble begins at r_{2e}. The expansion goes on till r_{2f} collapses to X_{r2f}^*. The martingale process kicks in after this and ends with sample period τ. Here we assume that the expansion duration of the first bubble is greater than that of the second bubble, so, $r_{1f} - r_{1e} > r_{2f} - r_{2e}$.

The data stamping process requires calculating $r_{1e,}$ r_{1f}, r_{2e} and r_{2f} from the following equations.

$$\hat{r_{1e}} = inf_{r2 \in [r0,1]}\{r_2 : ADF_{r2} > cv_{r2}^{\beta t}\} \tag{11}$$

and

$$\hat{r_{1f}} = inf_{r2 \in [\hat{r}_{1e + \frac{\log(T)}{T}}, 1]}\{r_2 : ADF_{r2} < cv_{r2}^{\beta T}\} \tag{12}$$

while

$$\hat{r_{2e}} = inf_{r2 \in \left[r_{1f}\ ,1\right]}\left\{r_2 : ADF_{r2} > cv_{r2}^{\beta t}\right\} \tag{13}$$

and

$$\hat{r_{2f}} = inf_{r2 \in [r_{2e + \frac{\log(T)}{T}}, 1]}\left\{r_2 : ADF_{r2} < cv_{r2}^{\beta T}\right\} \tag{14}$$

Then we use the backward sup ADF (BSADF) test to calculate the original and termination points based on the following equations.

$$\hat{r_{1e}} = inf_{r2\in[r0,1]}\{r_2: BSADF_{r2}(r_0) > scv_{r2}^{\beta t}\} \quad (15)$$

and

$$\hat{r_{1f}} = inf_{r2\in[\hat{r}_{1e+\frac{\log(T)}{T}},1]}\{r_2: BSADF_{r2}(r_0) < scv_{r2}^{\beta T}\} \quad (16)$$

while

$$\hat{r_{2e}} = inf_{r2\in[\hat{r_{1f}},1]}\{r_2: BSADF_{r2}(r_0) > scv_{r2}^{\beta t}\} \quad (17)$$

and

$$\hat{r_{2f}} = inf_{r2\in[\hat{r}_{2e+\frac{\log(T)}{T}},1]}\{r_2: BSADF_{r2}(r_0) < scv_{r2}^{\beta T}\} \quad (18)$$

One could sequentially apply this process detecting one bubble at a time, and then re-applying the same algorithm again and again. Once the first bubble has been detected, and it terminates at $r_{1f,}$ we use the equation below to date stamp the second bubble.

$$\hat{r_{2e}} = inf_{r2\in[\hat{r}_{1f+\in T},1]}\{r_2: \hat{r_{1f}}ADF_{r2} > cv_{r2}^{\beta t}\} \quad (19)$$

and

$$\hat{r_{2f}} = inf_{r2\in[\hat{r}_{2e+\frac{\log(T)}{T}},1]}\{r_2: \hat{r_{1f}}ADF_{r2} < cv_{r2}^{\beta T}\} \quad (20)$$

where $\hat{r_{1f}}ADF_{r2}$is the ADF statistic calculated over $(\hat{r_{1f}}, r_2]$. (8) We apply eq. (10) at the rate:

$$\frac{1}{cv^{\beta t}} + \frac{cv^{\beta t}}{T^{\frac{1}{2}}\partial_T^{\tau-\tau e}} \to 0, as\ T \to \infty\ eq. \quad (21)$$

Using the ADF detector, can identify the origin / termination of the first bubble, but not the second bubble, if the duration of the second bubble exceeds the first bubble, i.e., if $\tau_{1f} - \tau_{1e} > \tau_{2f} - \tau_{1e.}$ If the reverse is true, that is the duration of the first bubble is shorter than the second bubble, i.e., if $\tau_{1f} - \tau_{1e} < \tau_{2f} - \tau_{1e,}$ then under rate condition

$$\frac{1}{cv^{\beta t}} + \frac{cv^{\beta t}}{T^{1-\propto/2}} \to 0, as\ T \to \infty \tag{22}$$

this procedure can still detect the first bubble, but detects the second bubble with a delay as

$$(\hat{r_{2e}}, \hat{r_{2f}}) \xrightarrow{P} (r_{2e} + r_{1f} - r_{1e}, r_{2f}) \tag{23}$$

Under the BSADF methodology, we again apply eq. (10) at the rate of eq. (21). With continuous re-initialization, the BSADF detector can consistently estimate

$$(\hat{r_{1e}}, \hat{r_{1f,}}\, \hat{r_{2e}}, \hat{r_{2f,}}) \xrightarrow{P} (r_{1e}, r_{1f}, r_{1e}, r_{2f}) \tag{24}$$

of the origin and termination points of the first and second bubbles. Then under the sequential PWY methodology, using the same sequence of eq. (10) and rate eq. (21), we can consistently estimate in eq. (24), the origin and termination points of the first and second bubbles. Both the BSADF and the sequential PWY methodology, provide consistent estimates of the origin and termination of sequential bubbles. (9)

SECTION 3. EMPIRICAL APPLICATION

We apply the PSY [2014) methodology to identify for the presence of multiple bubbles in four major foreign i.e., non US financial indices, namely, the Financial Times Stock Exchange 100 Index, also called the “FTSE 100” or "Footsie", which is a share index of the 100 companies listed on the London Stock Exchange with the highest market capitalization, the benchmark French stock market index the CAC 40, which represents a capitalization-weighted measure of the 40 most significant values among the 100 highest market caps on the Euronext Paris, the DAX, which is a blue chip stock market index consisting of the 30 major German companies trading on the Frankfurt Stock Exchange, and the Nikkei 225, more commonly called the Nikkei index, which is a stock market index for the Tokyo Stock Exchange. [10]

We use monthly data for the FTSE 100 for the period December 1983 to November 2017, for a total of 408 observations; CAC for the period July 1987 to December 2017 for a total of 366 observations; DAX for the period December 1964 to November 2017 for a total of 636 observations; and the NIKKEI for the period April 1950 to December 2017 for a total of 813 observations. This data set was obtained from DataStream. The data used is the respective stock price index for the relevant month. We then conduct the SADF and the GSADF tests on the stock price index according to the basic model in eq. (1). The results are given in tables 1 - 4. Also given are the critical values of the two tests obtained from 2000 replications of the data in each case.

Table 1
FTSE 100

	Test Statistic	Finite Sample Critical Values		
Number of observations = 408		90%	95%	99%
SADF	1.8058	1.1423	1.4172	1.9799
GSADF	2.1761	1.9810	2.2173	2.7783

Table 2
CAC

	Test Statistic	Finite Sample Critical Values		
Number of observations = 366		90%	95%	99%
SADF	2.9994	1.1485	1.3784	2.0150
GSADF	3.0691	1.9324	2.1542	2.6512

Table 3
DAX

	Test Statistic	Finite Sample Critical Values

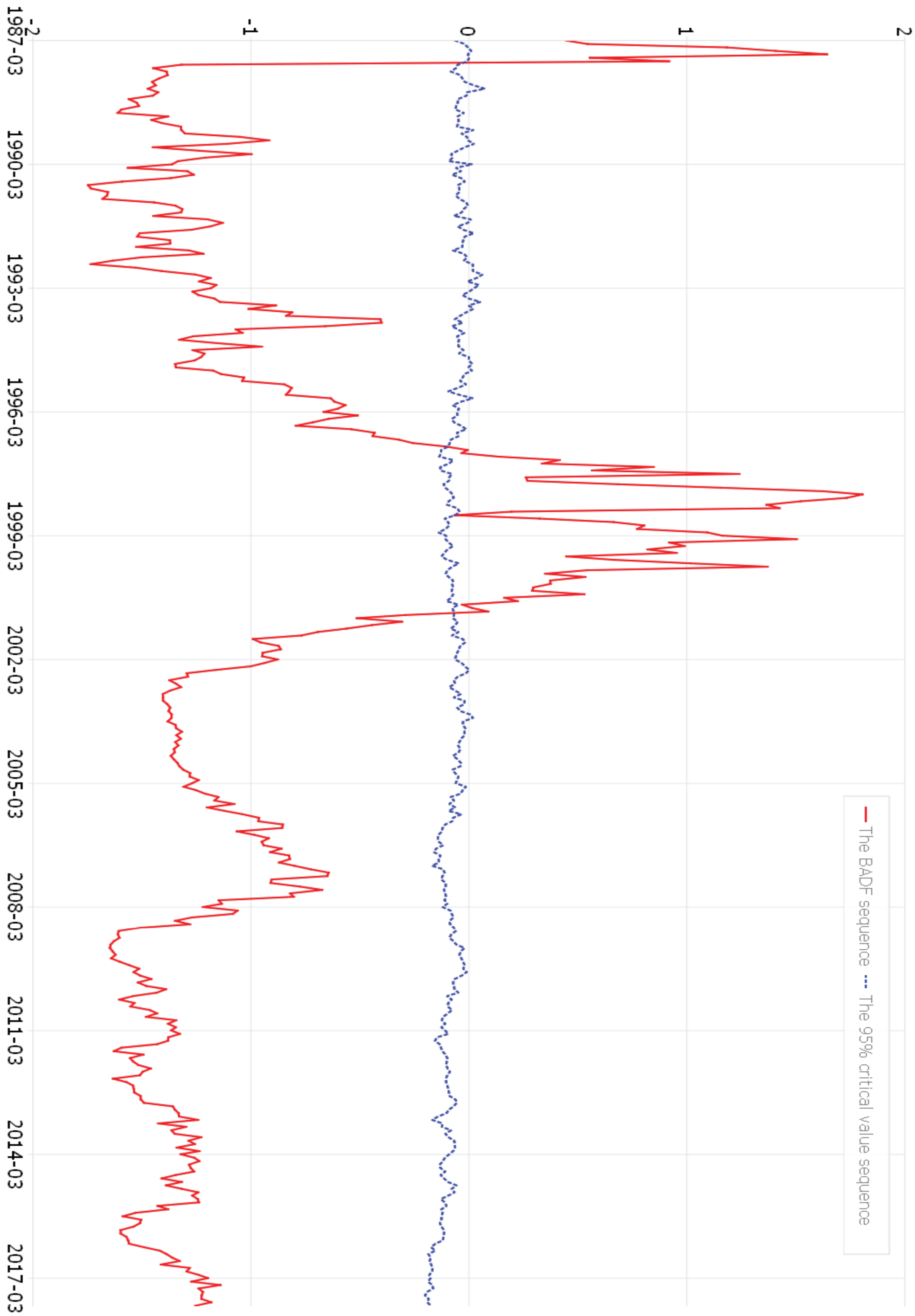
Number of observations = 636		90%	95%	99%
SADF	6.4794	1.2490	1.4934	2.0042
GSADF	6.4794	2.0746	2.2951	2.8082

Table 4
NIKKEI

	Test Statistic	Finite Sample Critical Values		
Number of observations = 813		90%	95%	99%
SADF	10.0832	1.2527	1.5407	2.0509
GSADF	10.0832	2.0745	2.2773	2.6917

Both tests find evidence of bubbles or explosive sub-periods over the long-term data in all 4 of the indices (test statistics in each case exceed the critical values for both test statistics considered). We then conduct a bubble monitoring exercise for each index using the backward ADF test and its critical value (using the PWY strategy), and the backward SADF statistic and its critical value (using the PSY strategy). This is done in graphs 1 – 8. In each graph the solid line represents the relevant test statistic, and the broken line represents the critical value. Figures 1, 3, 5, and 7 presents results from the use of the backward ADF test from the PWY paper, while figures 2, 4, 6, and 8 present results from the use of the backward SADF statistics from the PSY paper.

Figure 1 FTSE 100 Backward ADF statistic



In Figure 1 we look at the FTSE 100 and the existence of a bubble (test statistic greater than the critical value) is evident in the late 1990s to early 2000s, which corresponds with the technology bubble and its subsequent bursting. There is, however, no bubble around the financial crisis of 2008-09, which is surprising but in line with results that we have found in our study of U.S. indices.

Figure 2 FTSE 100 Backward SADF statistic

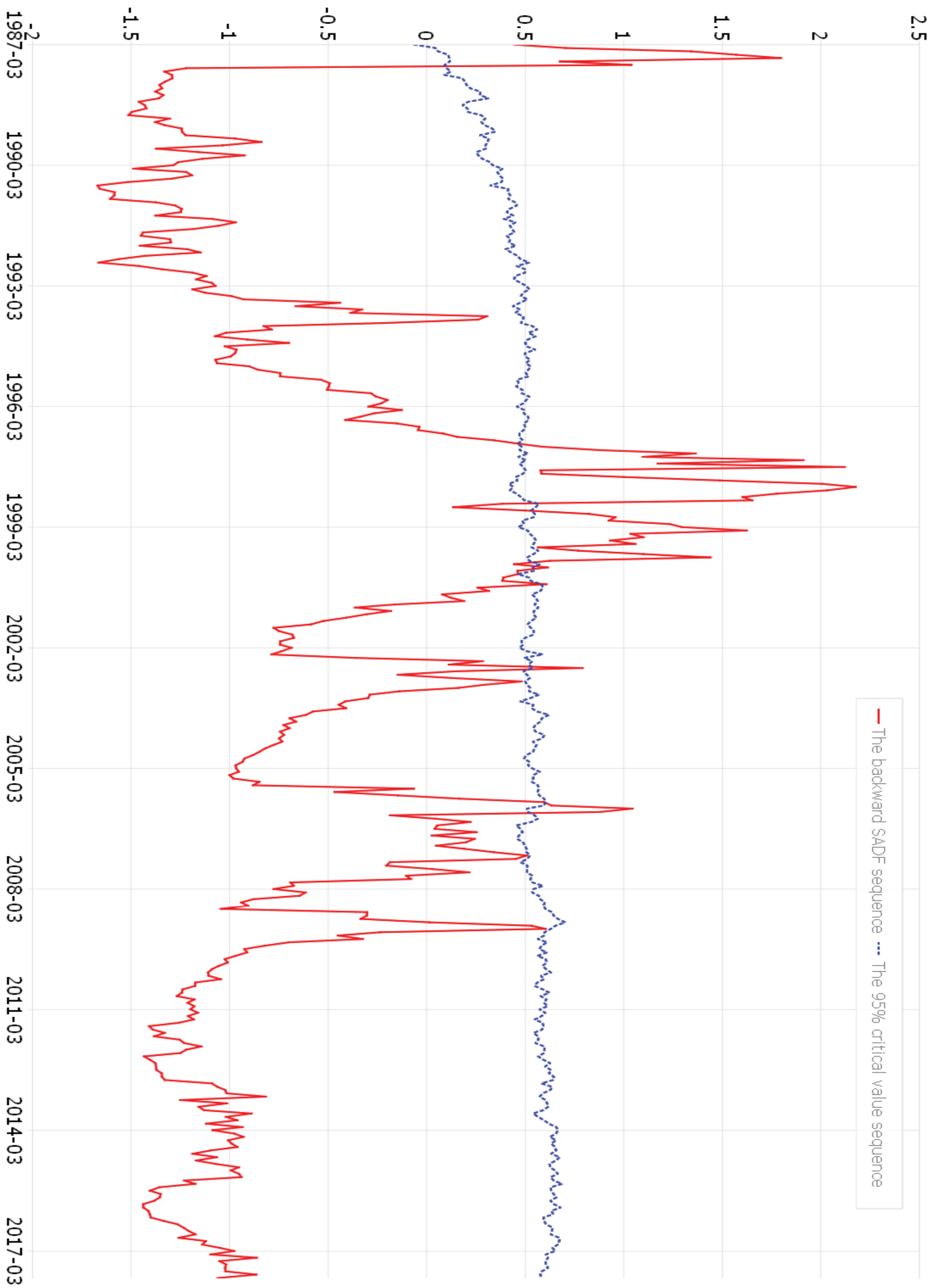


Figure 2 shows a bubble again for the late 1990s to early 2000s (just like in figure 1), but also seems to show bubbles (short ones) around 2003, 2006 and perhaps around 2007 and 2009. The ability of the BADF statistic to detect multiple bubbles is suspect, and therefore the results in Figure 2 (based on the PSY paper) are more reliable.

Figure 3 CAC Backward ADF statistic

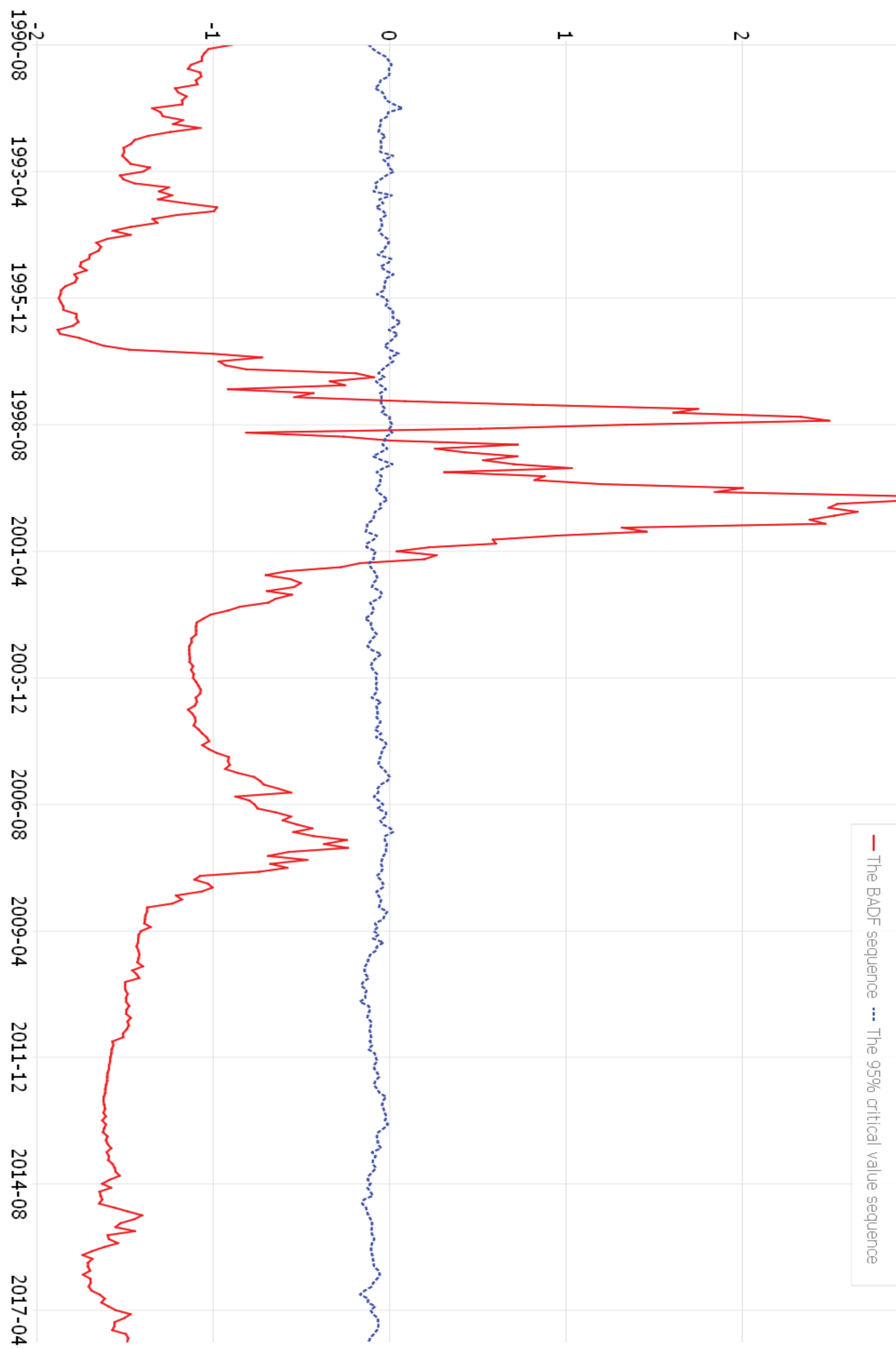
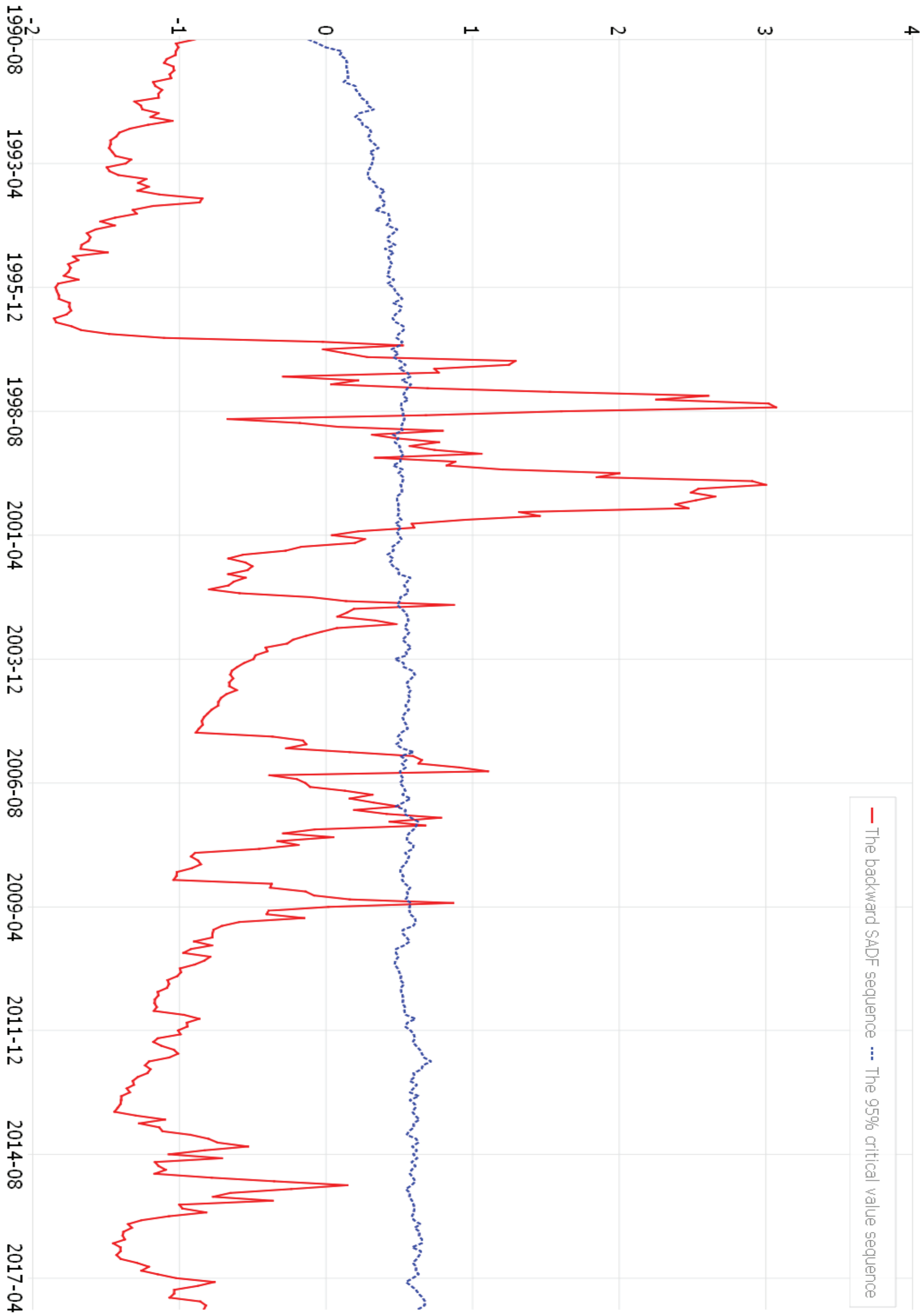


Figure 4 CAC Backward SADF statistic



A similar bubble monitoring exercise is carried out for the CAC index in figures 3 and 4. Figure 3 indicates a bubble around later 1990s to the early 2000s. Figure 4 indicates the existence of multiple bubbles for the CAC data. These bubbles occur in the late 1990s to early 2000s, also around 2002, 2005 and then again 2007, and then 2009.

Figure 5 DAX Backward ADF statistic

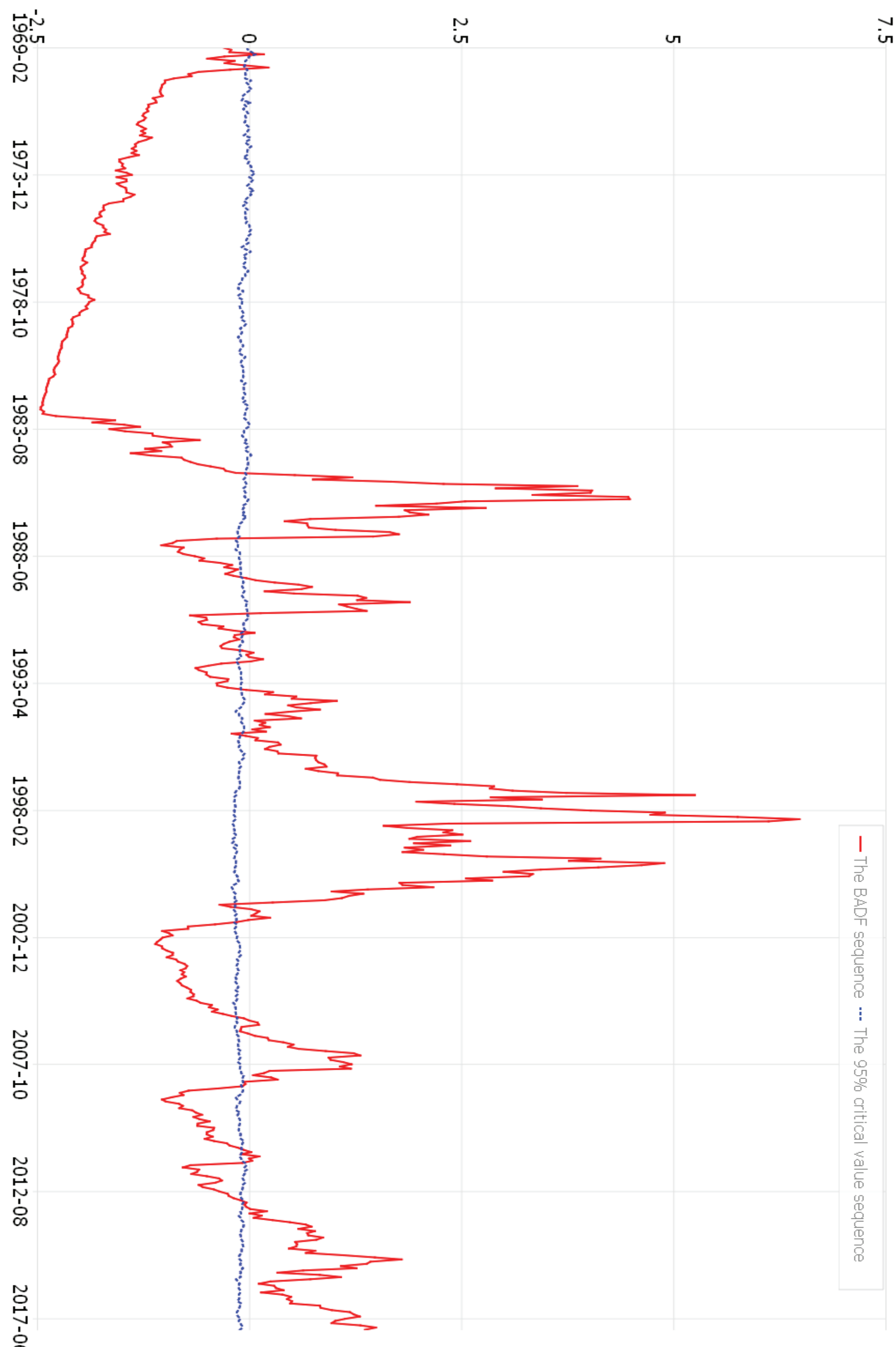
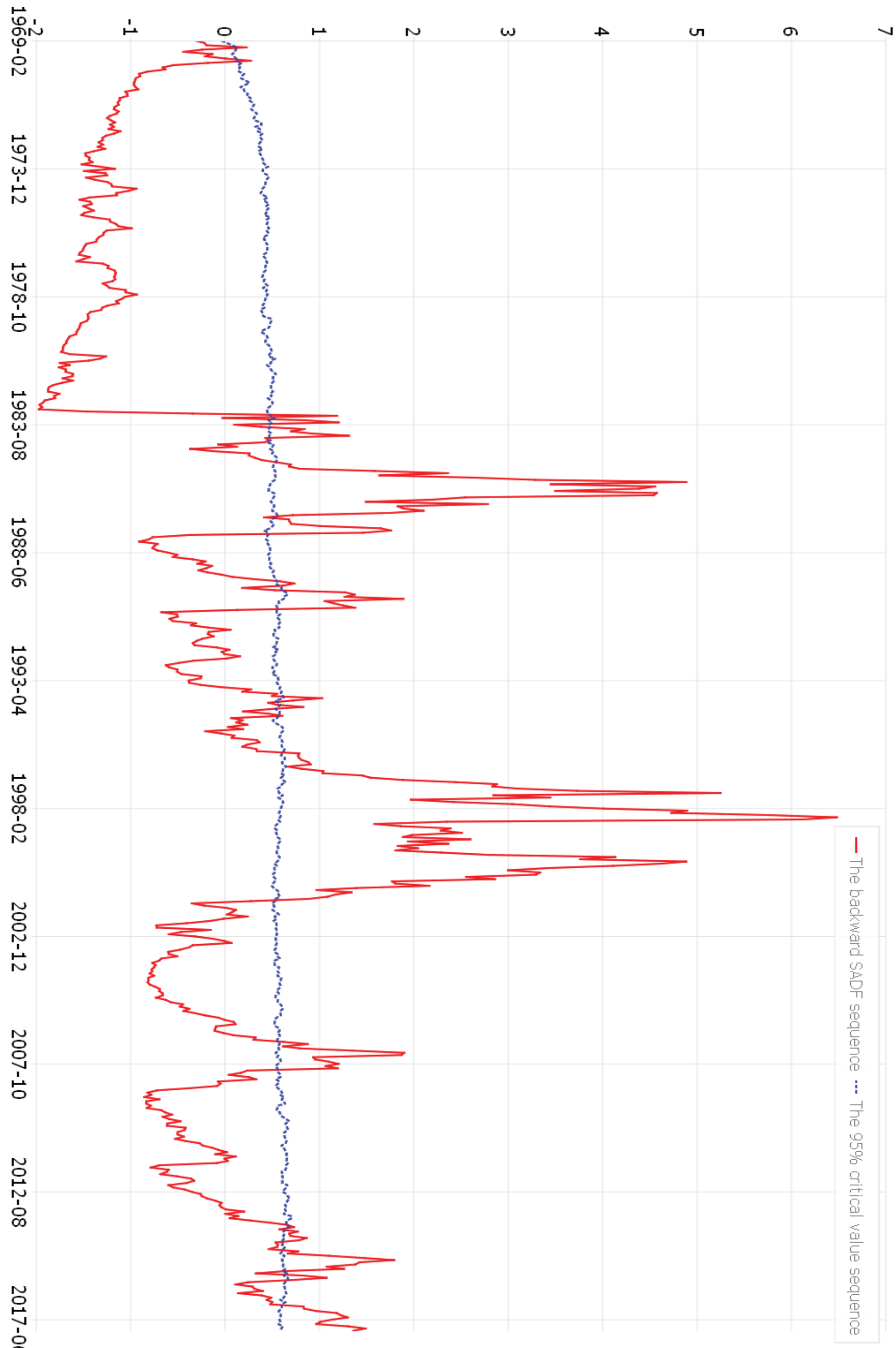


Figure 6 DAX Backward SADF statistic



Results for the DAX index are presented in figures 5 and 6. Figure 5 (BADF statistic) indicates the existence of bubbles in the mid and late 1980s, through most of the 1990s, then around 2007 and then again from about 2013 onwards. Figure 6 (backward SADF statistic) indicates very similar results for the DAX.

Figure 7 NIKKEI backward ADF statistic

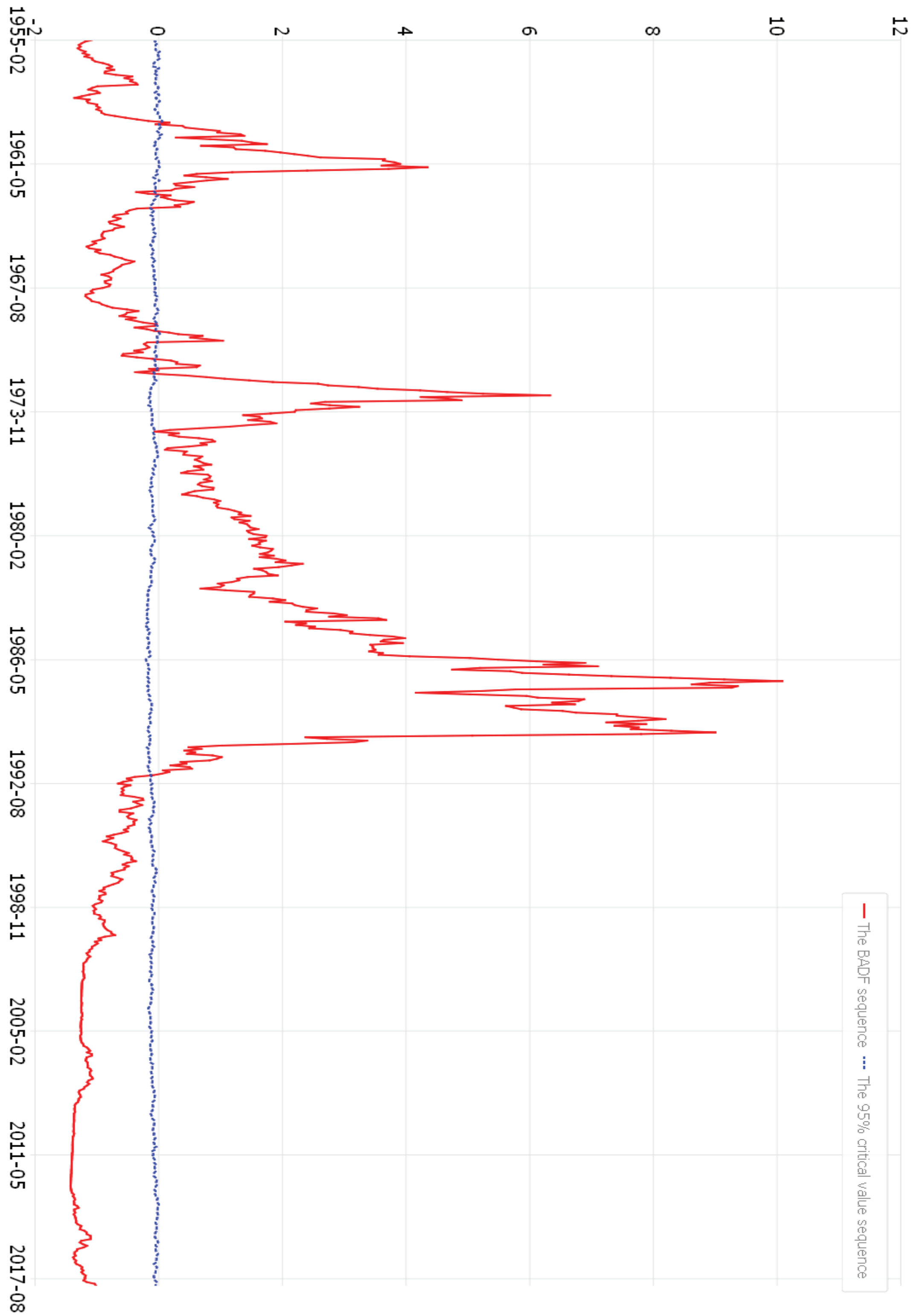
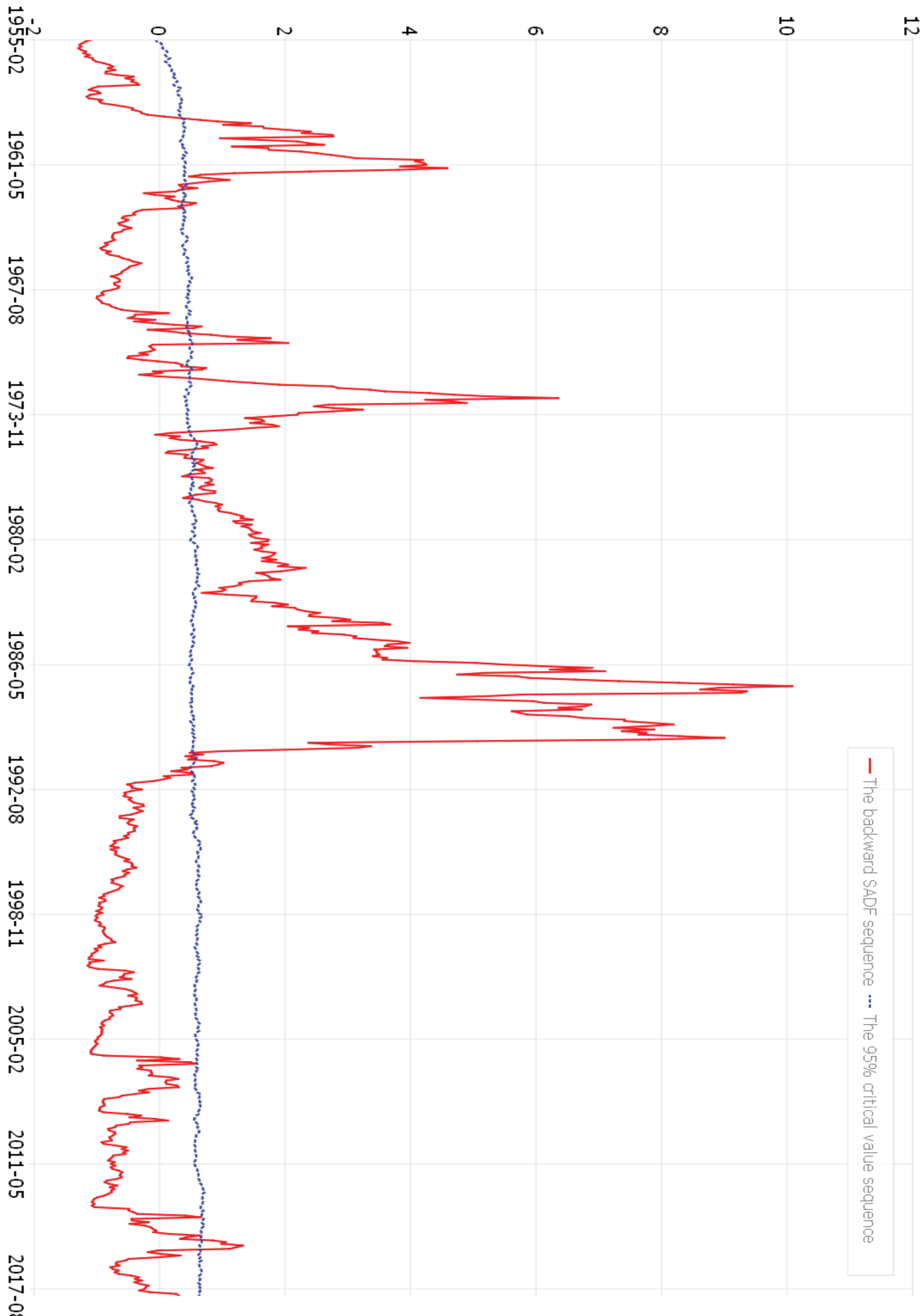


Figure 8 NIKKEI Backward SADF statistic



The NIKKEI index results are presented in figure 7 (BADF) and figure 8 (backward SADF). The span of the data for the NIKKEI index also happens to be the largest in our data set. Figure 7 indicates the existence of bubbles in the later 1950s and early 1960s, then from the mid-1970's all the way to 1992, and then no other bubbles including around 2008-09. Figure 8 shows very similar results for the NIKKEI.

Results from the backward ADF and the backward SADF statistic are quite different for the FTSE 100 and the CAC, both of which have data from the mid-to-late 1980s to 2017 and are quite similar for the DAX and the NIKKEI, for which we have data for much longer span (from 1964 (DAX) and from 1950 (NIKKEI). The backward SADF statistics (Figures 2, 4, 6, and 8) are considered more reliable for investigating multiple bubbles. We have data for the FTSE 100 and the CAC only from the 1980s, and this is clearly not enough to investigate whether there were bubbles in that time period. We have data for the DAX and the NIKKEI at least from the 1960s and these do indicate (in both cases) the existence of bubbles in the 1980s. All 4 indices have evidence for bubbles in the 1990s, up-to about the early 2000s, almost right when the technology bubble burst. We clearly do have proof of a bubble in the 1990s, and therefore we can conclude that there is evidence to support the widely referred "technology bubble" in the late 1990s and early 2000s. There is very limited evidence to indicate the existence of bubbles in 2007-2009, around the time of the financial crisis.

SECTION 4. CONCLUSION

The new test, the GSADF procedure is a recursive test, able to detect multiple bubbles. It's a rolling window, right sided ADF unit root test, with a double sup-window selection criterion. The SADF test is good, but it cannot credibly detect multiple bubbles over the same sample data set. The GSADF test overcomes this weakness and has significant discriminatory power in detecting multiple bubbles. It makes it very relevant in studying the "time trajectory" of long historical data sets. We have evidence for the existence of bubbles in the1990s for all 4 indices, thus providing evidence for the "technology bubbles" of the 1990s and early 2000s. There is limited evidence for bubbles in the 2000s and later, including around the time of the financial crises of 2008. This may indicate that the financial crisis, while its effects were felt worldwide and practically in all industries, may not have been a stock market bubble, but was a housing bubble which affected the stock market in many countries as the problems in the housing sector spread throughout the economy in many countries.

The technology bubble of late 1990s early 2000s (for which we do have evidence) was confined to the technology sector and did not spread to other sectors of the economy. Technology companies are directly part of stock indices and to that extent affect stock markets, but they do not (at least in the 1990s and 2000s) seem to have impacted the rest of the economy. The housing market seems to have had a far more significant and broader impact on the economy than the technology industry did, but the housing market does not seem to have caused a bubble in the stock markets worldwide.

Notes

1. Gurkaynak (2008) is a good review of this documentation.

2. DataStream proprietary data was purchased from EIKON, which was made possible due to a research grant of Professor Dutt, from the Richards College of Business, University of West Georgia. Our data is taken from DataStream International. ***As required by the IRBE journal, we are submitting the data set used in this research, but since it is proprietary, it can only be used to replicate our results, and is not for any other use, academic, research or otherwise.***

3. Sections 2 and 3 are a discussion of the PSY (2014) test, as implemented by us.

4. Cochrane (2005) debates the rationale of including "bubble components" in an asset pricing model, while Cooper (2008) expresses bewilderment at the literatures attempt to rationalize the well accepted NASDAQ bubble, as an accurate reflection of the changing market times and environment.

5. Interestingly, the experts agree more on the presence of market exuberance leading to panics, either rationally or irrationally. It's based on changing economic fundamentals, arising from behavior alterations of market players, or due to changing discount rates over time etc.

6. Then there is the Markov-switching test of Hall, et.al (1999), to detect explosive behavior in the data sample, but it is open to suspicion since Shi (2013) found it to be susceptible to "false detection of explosiveness." Also, according to Funke et.al. (1994) and van Norden and Vigfusson (1998), general filtering algorithms cannot differentiate between spurious explosiveness (the marker being high variance) as opposed to generic explosive behavior. The general approach of SADF is also used by Busetti and Taylor (2004) and Kim (2000) among others, to study "market bubbles" but the simulation study done by Homm and Breitung (2012) finds the PWY (SADF) test to be the most powerful metric in detecting multiple bubbles.

7. We briefly outline the two cases of no bubbles, and a single bubble. PSY (2015) develops the limit theories and consistency properties in case of single and multiple bubbles. PSY (2015, b) is a supplement describing the robustness checks of this testing procedure. If the null is of no bubbles, i.e., the data path is a normal growth trajectory, the ADGF and the SADF (extracted from Theorem 1) is that the backward ADF is nothing but the special case of GSADF when $r_1 = 0$, with fixed r_2, while the backward SADF is the special case of the GSADF test with $r_1 = (r_2 - r_w)$ and fixed r_2. Based on the limit theorem, the advantage here is that under the null of "no bubbles" the chances of a false positive (spurious detection) of a bubble origination and termination using backward ADF statistic and the SADF statistic, tends to zero and so, $P_r \{\hat{r}_e \{ \hat{r}_e \in [r_0, 1]\} \rightarrow 0$, and $P_r \{\hat{r}_f \in [r_0, 1]\} \rightarrow 0$. But if single bubble episode is studied using this reduced form equation:

$X_t = X_{t-1}1\{t < r_e\} + \partial_t X_{t-1}1\{r_e \leq t \leq r_f + \left(\Sigma_{k=rf+1}^{t} \epsilon_k + X_{rf}^{*}\right)1\{t > r_f\} + \epsilon_t\ 1\{j \leq r_f\}$ eq. (25)

to detect a martingale behavior, with the genesis of an explosion (or birth), its existence and eventual collapse from origin to renewal of the subsequent behavior. In equation (25), $\delta_t = 1+cT^{-\alpha}$ with $c>0$ and $\alpha \in (0, 1)$ and error ϵ_t is iid $(0, \sigma^2)$, and $X^{*}_{rf} = X_{re}+X^{*}$ with $X^{*} = O_p(1)$. Here r_e =

$[Tr_e]$ dates the origin of the bubble expansion while $r_f = [T_{rf}]$ dates the collapse of the bubble. The bubble expansion period is given by $B = [r_e, r_f]$, with the expansion rate given by the autoregressive coefficient δ_t. Even mildly explosive features capture the market exuberance quiet well. Over time the bubble collapses to X^*_{rf} and then follows a standard martingale over the subsequent period $N_1 = (r_f, T)$. This equation captures single bubbles very well, as demonstrated by the PSY (2014.) Their simulations show that under eq. (25) both ADF and BSADF provide consistent estimates of the origination and termination dates of a single bubble episode.

8. This sequential procedure (for proper and credible application) requires a long set of observations, the longer the better, in order to re-initialize the test process after a bubble.

9. Other conclusions PSY (2014) arrived at are:

    1. In case of a single bubble, the power of PWY, sequential PWY and CUSUM are the same, but less than GSADF.

    2. As the bubble duration increases, so do the power of these tests.

    3. In case of a single bubble, the PWY and CUSUM are reasonably accurate, but the sequential PWY tests tend to overestimate the bubble number.

    4. In case of two bubbles, the outcomes are mixed. Here the bubble duration becomes an important variable, and the possibilities are:

        a) If the first bubble is larger than the second bubble, PWY underestimates the numbers, because in most cases it identifies the first bubble but not the second one.
        b) If the second bubble duration is larger than the first, the PWY test can more confidently detect both bubbles. It is the same for the CUSUM test. The sequential PWY performs as well as the PSY test.

10. This is a part of a bigger project, where we are examining for the presence of multiple bubbles and exuberance episodes, using the major USA financial indexes like the DOW, NASDAQ and the S&P 500. In an attempt to make the testing more comprehensive and worldwide, here in this extension, we examine four major foreign indexes.

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