Idiosyncratic Risk and Returns: The Case for a More Efficient Class of Estimators

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Abstract

Volatility is a key input into many important financial decisions. Therefore, accurate forecast of volatility plays an important role in making these decisions. Typically, volatility is forecast using realized volatility computed from closing stock prices. Employing expectation of volatilities such calculated, several papers find that expected idiosyncratic risk is positively associated with contemporaneous returns. Yang and Zhang (2000) show that estimators belonging to the class of range-based estimators are more efficient than the estimators derived only from closing prices. Using the more efficient range-based volatility estimates, we find no evidence to support the hypothesis that idiosyncratic risk explains returns.

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Introduction

Volatility plays an important role in key financial decision including portfolio choice, pricing of derivatives and other financial assets, and risk management. Therefore, precise measurement of realized volatility and accurate forecasts of conditional volatility become crucial for these financial decisions. Since the publication of Goyal Santa-Clara (2003) (hereafter, GSC) which showed that contrary to the prevailing notion in finance, idiosyncratic risk is priced by the market, estimation of realized and conditional volatility has received a lot of attention in the finance literature. Although several recent papers employing the US and international data also arrive at the conclusion reached by GSC, some authors find mixed results about the relationship between conditional volatility and return.

Broadly, research in this stream of finance has explored two interconnected issues: the estimation of realized volatilities; and translating realized volatilities into expectations of future volatilities. Our paper adds to this debate by analyzing a sparsely-used (in finance) class of volatility estimators based on enhanced information than just close-to-close returns. The estimators we present belong to the class of range-based estimators which have been shown to be more efficient than the estimators based on closing prices (see Garman and Klass (1980) and Yang and Zhang (2000) among others). We employ two range-based estimators based on daily high, low, open, and close prices to find that contrary to the evidence documented in recent papers, no relationship exists between idiosyncratic risk and stock returns.

The classical finance theory is based on the idea that risk is positively associated with future returns, and that the only risk that matters is the systematic risk, commonly represented by beta. For example, the Capital Asset Pricing Model (CAPM), a well-known asset pricing model predicts that the future return of a stock depends on the stock's market beta. In CAPM, idiosyncratic risk ceases to matter because it can be diversified away. However, the assumption that investors are adequately diversified has faced challenges from several empirical and theoretical papers which argue that investors may remain under-diversified for a variety of reasons. Levy (1978) lists studies which show that individual investors are highly undiversified. More recently, Goetzmann and Kumar (2008) find that individual investors in the US are under-diversified and the level of under-diversification is higher for younger, low-income, less-educated, and less-sophisticated investors. These studies cast doubts about the notion that idiosyncratic risk is diversified away and should not be priced.

Our paper addresses two related issues—the impact of idiosyncratic risk on returns; and the measurement of idiosyncratic risk. To that end, we next present a review of the literature in the two areas. In the review, we first describe the research which shows that the use of a larger set of information than just closing prices yields more efficient estimators of realized volatilities. Second, we describe the state of theoretical and empirical research in the area of idiosyncratic risk and its effect on return.

Measurement of Volatilities

Stock returns are computed using closing prices. The often used measures of realized volatilities over a period (say a month) take the standard deviation of residuals of close-to-close returns obtained from a pricing model as a proxy for realized volatilities. Since the variance of a close-to-close estimator depends on the inverse of the number of observation during the estimation interval, it is possible to reduce the dispersion by making use of higher frequency data (Andersen, Bollerslev, Diebold, and Labys (2003)). But when available, the higher frequency data suffers from market microstructure problems. If the higher frequency data cannot be obtained, it is pertinent to ask the question whether a more efficient estimator can be found by inclusion of more information than just the closing prices.

Garman and Klass (1980) is perhaps the earliest attempt at incorporating open, high, and low prices beside the close prices into estimation of volatilities. They show that the estimator derived using more information has a variance markedly lower than that of the classical estimator based on close-to-close prices. However, the Garman and Klass estimator is not independent of the drift and opening jumps in stock prices. To take into account drift in stock prices, Rogers and Satchell (1991) proposed a drift-independent model based on multiple price points during a trading day. But Rogers and Satchell (1991) corrects only for the drift and does not account for opening jumps. Yang and Zhang (2000) develop a minimum-variance estimator which is independent of both drift and opening jumps. In this paper, we use Rogers and Satchell (1991) and Yang and Zhang (2000) estimators of realized volatilities to conduct our analyses.

Idiosyncratic Risk and Returns

Mayers (1976) explores the effect of nonmarketable assets and market segmentation on asset prices. In his model, Mayers finds that under the assumption of constant relative risk aversion less than or equal to one, asset prices are lower given nonmarketable assets and market segmentation. In Mayers (1976) each investor holds a unique portfolio contrary to the prediction from CAPM. Levy (1978) allows investors to hold portfolios with some given number of securities. He finds that individual stock variance is important in his model. Merton (1987) models capital market equilibrium in an incomplete information setting and finds that less well-known stocks with fewer investors will tend to have larger expected returns and that expected returns depend on both the market risk and the total variance. Campbell, Lettau, Malkiel, and Xu (2001) list several arguments for the importance of idiosyncratic risk to expected returns. These arguments include: a lack of investor diversification from not following the approach recommended by financial theory or due to constraint imposed by compensation policy; investors may diversify by holding a portfolio of thirty stocks or fewer which depending on the volatility of individual stocks may not be adequate; arbitrageurs who exploit mispricing of individual securities are exposed to idiosyncratic risk; idiosyncratic volatility becomes important in event studies; and option price on a stock depends on total volatility of returns which is made up of volatilities attributable to both the market and to a specific firm. And Malkiel and Xu (2006) present a model in which if a group of investors does not hold the market portfolio, remaining investors will also not be able to hold the market portfolio and idiosyncratic risk may become important.

Turning attention to the empirical treatment of the issue, several papers show that the relationship between idiosyncratic risk and expected returns is either positive, or non-existent, or even negative. These studies are based on US data and use monthly intervals. French, Schwert, and Stambaugh (1987) find a positive relationship between the expected risk premium on common stocks and predictable level of volatility. Lehmann (1990) finds that the residual risk has a significant co-efficient when he corrects for problems in the statistical methods used in prior studies. In a recent paper, GSC show that average monthly stock variance is positively associated with higher returns in the subsequent month. Fu (2009) uses the exponential GARCH models to estimate expected idiosyncratic volatilities and finds a positive relationship between the conditional idiosyncratic volatilities and expected returns. Malkiel and Xu (2006) control for factors like size, book-to-market, and liquidity in conducting their analyses for US and Japanese equities to find that idiosyncratic volatility is more important than either the β, the systematic risk, or the size in explaining the cross-section of returns. Huang, Liu, Rhee, and Zhang (2009) also document a positive relationship between conditional idiosyncratic volatility and expected returns.

Estrada (2000) uses a database of 28 emerging economies and finds that idiosyncratic risk is significant in explaining the cross-section of returns. Harvey (2000) uses data from 47 different countries to construct 18 different measures of risk. He finds that collectively idiosyncratic risk is positive in explaining the cross-section of expected returns. Brockman, Schutte, and Yu (2009) examine the relationship across 44 countries from 1980 to 2007. They find a significantly positive relationship and attribute it to under-diversification. Lee, Ng, and Swaminathan (2009) obtain data for G-7 countries over the 1990 to 2000 time period and find a positive relationship between idiosyncratic volatility and expected returns.

Although the evidence in favor of a positive relationship between idiosyncratic risk and returns seems dominant, some papers document conflicting results. Longstaff (1989) observes a consistently negative but insignificant relationship between variance and returns for the overall period 1926-1985 and for the three sub-periods in which he divides his sample. Bali, Cakici, Yan, and Zhang (2005) re-examine the relationship between average stock volatility and future returns to conclude that the results in GSC were driven because of small stocks traded on the NASDAQ and that the GSC results disappear when market values are used as weights instead of equal weights to compute average volatility. And Wei and Zhang (2005) find that the results in GSC are driven mainly by the data in the 1990s as the relationship between idiosyncratic risk and future returns disappears when they extend the sample to 2002. Wei and Zhang also raise the possibility that combining equally-weighted average volatility with value-weighted average return may be behind the results reported in GSC. Bali and Cakici (2008) employ a portfolio approach and use various different measures of idiosyncratic volatility, alternative weighting schemes, different breakpoints for the construction of portfolios, and two different samples to find no robust relationship between idiosyncratic volatility and expected returns. Finally, Ang, Hodrick, Xing, and Zhang (2006) find that stocks with high idiosyncratic volatilities have low average returns, which is the opposite of that documented in GSC.

Therefore, the overview of the literature on the relationship between idiosyncratic risk and returns has not been settled as different papers have reported mixed results. In the next section we describe the two methods used in our paper to measure realized volatilities which will be used to estimate conditional volatilities.

Measures of Realized Volatility

Close-to-Close Volatility

Use method of Goyal Santa-Clara and others. Removed firms with less than 15 daily returns in a month. Some firms have missing returns in a month. Therefore, for each firm, the computed standard deviation is converted into a monthly standard deviation by multiplying by the square root of number of observations divided by the number of trading days in a month.

Fama-French Three Factor Method

In this approach, the idiosyncratic volatility for a stock in a month is computed as the standard deviation of residuals from the regression of daily excess returns on the daily Fama-French (1993, 1996) three factors in that month. Ang, Hodrick, Xing, and Zhang (2008) and Fu (2009) use this approach. Thus in a given month, we run the following regression for each stock *i* for days 1 through *n* in that month,

$$r_{it} - r_t = \alpha_{it} + \beta_{it}(r_{mt-}r_t) + s_{it}SMB_t + h_{it}HML_t + \varepsilon_{it}. \qquad (2)$$

The realized monthly volatility (*VAFF*) from equation (2) is the standard deviation of the error terms, ε_{it} multiplied by the square root of the number of trading days, *n*, in the month.

Range-Based Methods

Our first range-based measure uses Rogers and Satchell (1991) and Rogers, Satchell, and Yoon (1994). If O_i, H_i, L_i, and C_i are the open, high, low, and close prices respectively for a stock on day i, $C0_i$ is the closing price for the stock on the previous day, and *n* is the number of trading days in a month then,

$$o_i = \ln(O_i) - \ln(C0_i),$$

$$u_i = \ln(H_i) - \ln(O_i),$$

$$c_i = \ln(C_i) - \ln(O_i),$$

$$d_i = \ln(L_i) - \ln(O_i),$$

and,

$$V_{RS} = \frac{1}{n}\sum_{i=1}^{n}[u_i(u_i - c_i) + d_i(d_i - c_i)]. \qquad (3)$$

V_{ARS}, the realized volatility from equation (3) in a given month is then computed as $V_{RS}\sqrt{n}$.

The second range-based measure of volatility (V_{AYZ}) is due to Yang and Zhang (2000) and calculated as follows,

$$V_{yz} = V_0 + kV_c + (1-k)V_{RS}. \qquad (4)$$

Where,

V_{RS}=Volatility calculated using Rogers and Satchell (1991) and Rogers, Satchell, and Yoon (1994) and,

$$V_O = \frac{1}{n-1}\sum_{i=1}^{n}(o_i - \bar{o})^2,$$

$$V_c = \frac{1}{n-1}\sum_{i=1}^{n}(c_i - \bar{c})^2,$$

$$\overline{O} = \frac{1}{n}\sum_{i=1}^{n} o_i,$$

$$\overline{C} = \frac{1}{n}\sum_{i=1}^{n} c_i.$$

V_{AYZ}, the realized volatility using equation (4) in a given month is then computed as $V_{YZ}\sqrt{n}$.

Fu (2009) argues that the relationship between idiosyncratic risk and returns is contemporaneous. He also finds that idiosyncratic volatility varies substantially over time which would indicate that using realized volatilities to test the relationship may not be appropriate. Therefore, we estimate expected volatilities using realized volatilities based on the approaches described next.

Specifications of Conditional Volatility

The EGARCH Model

We estimate the conditional idiosyncratic volatility for a stock using two different approaches. To forecast volatility in month *t*, we first obtain the monthly residuals, u_t, for a stock by employing equation (2) in the months from the beginning of the sample period to the month *t-1*. The EGARCH (*p*,*q*) model is then used to forecast volatility in month *t*. Following Fu (2009), we vary *p* and *q* from 1 through 3 and obtain 9 different models. From among the models that converged in a month, we choose the best-fit model as the one with the lowest Akaike Information Criterion.[1] The specification of the conditional variance of u_t is:

$$\ln(h_t) = \omega + \sum_{i=1}^{q} \alpha_i g(z_{t-i}) + \sum_{j=1}^{p} \gamma_j \ln\left(h_{t-j}\right).$$

Where,

$$g(z_t) = \theta z_t + \gamma[|z_t| - E|z_t|],$$

and

$$z_t = \frac{u_t}{\sqrt{h_t}}.$$

In estimation, the parameter γ is assumed to be one and $E|z_t| = \sqrt{\frac{2}{\pi}}\ if\ z_t \sim N(0,1)$.

[1] We get similar results if we choose the best-fit model using the Schwarz Bayesian criterion.

$\sqrt{\exp(\ln(h_t))}$ is our first estimate of the conditional volatility, *VEGFF*.

The ARIMA Model

We employ ARIMA (p,q) to get our next two estimates of conditional volatility using realized volatilities based on Rogers and Satchell (1991) and Yang and Zhang (2000). The equation for forecasts of volatilities takes the form:

$$\sigma_t = \epsilon_t + \alpha_1\sigma_{t-1} + \cdots + \alpha_{t-p} + \theta_1\epsilon_{t-1} + \cdots + \theta_q\epsilon_{t-q}.$$

For every stock in the 30-month period prior to month t, σ is computed using equation (3) or (4). We restrict forecasting of volatility in the ARIMA approach to only those stocks that have at least 24 monthly returns in the 30-month window. To arrive at the best-fit model, we used twenty five different specifications by varying p and q from 1 through 5. Out of the 25 models, we retain only those which converged. The best-fit model is selected from among the ones that converged based on the minimum Schwarz criterion. The forecasts from the best-fit model provide us our second and third measures of conditional volatility, *VEARS* and *VEAYZ*.

Data and Variables

We use the daily and monthly CRSP data for the market information and the Compustat database for the book value of equity. The daily and monthly three factors are downloaded from the website of Kenneth R. French.[2] Since open prices are available for the NYSE/AMEX/NASDAQ firms only from June 15, 1992, onward, and we need 24 months of data to estimate the ARIMA models, our sample is limited to the period between June, 1994 and December, 2015. When daily market data over a month is required, we follow Fu (2009) and impose the restriction of a minimum of 15 days in a month for which a stock must have both a return and a non-zero trading volume.[3]

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[2] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

[3] In the September of 2001 there were only 15 trading days. Therefore, in that month, we exclude a firm from our sample if it did not meet the inclusion criteria for at least 12 trading days.

To compute range-based estimators. If vol=0 then delete. If prc=. Or prc lt 0 or askhi=. Or askhi lt 0 or bidlo=. Or bidlo lt 0 then delete. If previous close=open=hi=low=close then delete. If u, d, c=0 then delete.

"Daily trading prices for the NASDAQ National Market securities were first reported November 1, 1982. Daily trading prices for The NASDAQ SmallCap Market were first reported June 15, 1992. Therefore, Bid or Low Price for NASDAQ securities is always a bid before these dates." This paragraph from CRSP documentation.

In our cross-sectional regressions, we include several control variables. Fama and French (1992) show that the book-to-market ratio (*BM*) and firm size (*ME*) are useful in explaining cross-sectional returns. Jegadeesh and Titman (1993) demonstrate that buying past winners and selling past losers generates significantly positive returns over the horizons between three and twelve months. Following Fu (2009), for a stock in a month, we include the cumulative returns (*CRET*) in the six month period ending two months prior to the month as an explanatory variable. Consistent with Chordia, Subrahmanyam, and Anshuman (2001), to capture liquidity and its variability, we introduce the turnover ratio (*TURN*) defined as the natural log of the number of shares traded in a month divided by the number of shares outstanding expressed as percentage and the natural log of the variability of the turnover ratio, defined as the coefficient of variation of the turnover ratio (*CVTURN*), in our regressions. We impose a restriction of at least 18 observations in the computation of the two turnover related variables. Additionally, we follow Anderson and Dyl (2005) rule of thumb and adjust the NASDAQ volume down by 50 percent before 1997 and 38 percent after 1997 to address the effect of double-counting of trading volume for firms listed on that exchange. Finally, we include the systematic risk (*BETA*) of a stock in our cross-sectional return model.

BETA, *BM*, and *ME* are computed using the Fama French (1992) approach. Specifically, in June of every year, stocks are sorted into 100 size (number of shares times the stock price at the end of previous December) and pre-ranking beta portfolios. The pre-ranking betas are obtained from the regression of excess stock returns on the value-weighted market returns over the past 60 months. A minimum of 24 monthly returns are required for the estimation of pre-ranking betas. For the 100 size-beta portfolios, simple average returns are calculated from July of that year to June of the next year: based on portfolios constructed in June of year *y*, we have 100 portfolios returns in each month from July of year *y* to June of year *y+1*. This procedure is repeated every June over the sample period (1994-2015). For each portfolio, we run a regression of its monthly returns on the monthly-value-weighted market returns and its lag for the entire sample period. The portfolio beta is the sum of the coefficients on the value-weighted market return and its lag. Finally, each stock is allotted the beta of the portfolio in which it resides in June as a proxy for the systematic risk, *BETA*. *BM* is calculated as the natural log of the book-to-market ratio. In June of each year *y*, the book value of equity is obtained from the fiscal-year end statement of year *y*-1. The market value of equity is obtained from stock prices at the end of December in year *y*-1. The same *BM* is used for all the months between July of year *y* and June of year *y*+1. *ME* is computed as the natural log of the market capitalization in June of year *y* to explain returns from July of year *y* through June of year *y*+1.

Results

In Table 1, we present the autocorrelations for the three methods of realized volatilities. The autocorrelations for each firm are computed at various lags and then averaged across the sample firms. *VAFF*, the realized volatility based on closing prices decays relatively more quickly over the first three lags and then very slowly for higher lags. The more efficient realized volatilities based on information about high, low, open, and close prices (*VARS* and *VAYZ*) decay more quickly for four lags and appears to be persistent for lags greater than four.

Table 2 describes our variables of interest. All the variables are winsorized at 0.5 percent in each tail. We also exclude observations with monthly returns of greater than 300 percent to minimize the possibility of recording errors contaminating our results. The mean and median realized volatilities using the range-based estimators (*VARS* and *VAYZ*) are higher than those using closing prices (*VAFF*). Mean and median volatility forecasts (*VEGFF*, *VEARS*, and *VEAYZ*) also show a similar pattern. Other variables are comparable to the numbers reported in Fu (2009) in terms of means and medians. *VAFF* and *RET* exhibit right skewness of more than 3, but the other variables do not appear to be highly skewed.

Sample correlations among our measures of realized volatilities, conditional volatilities, and returns contemporaneous with conditional volatilities are available in Table 3. Although realized volatilities are strongly correlated, the correlations between closing-price-based conditional volatility (*VEGFF*) and range-based volatilities (*VEARS* and *VEAYZ*) are relatively weaker. As expected, the two measures of range-based conditional volatilities are highly correlated. In the univariate analysis, the correlation between *VEGFF* and *RET* is insignificant, but significant and negative between *VEARS* and *RET* and *VEAYZ* and *RET*.

In Table 4, we present our main result to test the hypothesis that there is a relationship between idiosyncratic risk and returns. In our cross-sectional regressions, the *t*-statistics are based on the Fama and MacBeth (1973) approach. Using all the sample firms, we run the cross-sectional regression with monthly stock returns as the dependent variable each month and generate a time series of monthly parameter estimates. From the time series of parameter estimates, we compute the mean estimate and the standard deviation of the estimate to calculate the *t*-value.

We present nine specifications of the return model. In the first specification, *BETA*, *BM*, and *ME* do not seem to be helpful in explaining returns. The explanatory power of the model given by r-square is also small (3.5 percent). We then introduce *CRET*, *TURN*, and *CVTURN* to the model. *CRET* and *CVTURN* are significant and the explanatory power of the model goes up to 17.71 percent.

Then we introduce our three measures of realized volatilities one by one into the return model with the seven exogenous variables. As in GSC, *VAFF*, *VARS*, *and VAYZ* are the naïve forecasts of conditional volatility. To wit, *VARS* realized in the month *t*-1 is the forecast of conditional volatility in the month *t*. *VAFF* and *VAYZ* are negative at 5-percent level or better, *VARS* is not.

To get the main results of our paper, we finally include the three measures of conditional volatilities, *VEGFF*, *VEARS*, and *VEAYZ*. Consistent with Fu (2009) and Huang et al. (2010), we find the closing-price-based volatility forecast, *VEGFF*, to be positive related to returns. However, the more precise measures of volatility, *VEARS* and *VEAYZ* are not significant in explaining returns and lead us to conclude that there is no relationship between idiosyncratic risk and returns.

Conclusion

Classical asset pricing theories posit no relationship between the idiosyncratic risk and returns. Research shows that the prediction may not hold true for a variety of reasons including a lack of adequate diversification on part of the investors. Nonetheless, empirical papers adopting different methodologies show that the relationship between idiosyncratic risk and returns is either positive, or nonexistent, or even negative. In any test of the relationship, the estimate of conditional volatility is the main ingredient. The classical estimators of realized volatility, which is used to forecast future volatility, are based on closing stock prices and have been shown to be highly imprecise. We adopt two estimators of realized volatility from the class of range-based estimators shown to be much more efficient than the classical estimators and use them to forecast volatility. Contrary to recent papers, we find no evidence of a relationship between idiosyncratic risk and returns.

Our paper uses methodologies used in existing research to estimate conditional volatilities. Future research may explore the issue of relative merits of different methodologies used to forecast volatilities.

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Table 1
Autocorrelations for the Three Realized Volatility Measures

Variable	LAG1	LAG2	LAG3	LAG4	LAG5	LAG6	LAG7	LAG8	LAG9	LAG10	LAG11	LAG12
VAFF	0.33	0.27	0.25	0.19	0.18	0.17	0.15	0.14	0.15	0.12	0.10	0.12
VARS	0.45	0.35	0.29	0.23	0.21	0.19	0.17	0.17	0.16	0.15	0.12	0.13
VAYZ	0.39	0.30	0.26	0.20	0.19	0.17	0.15	0.15	0.15	0.13	0.10	0.11

Table 2
Summary Sample Statistics

Variable	N	Mean	Median	Skew	Q1	Q3
VAFF	1,678,344	0.12	0.09	7.76	0.05	0.16
VARS	1,783,010	0.13	0.09	1.95	0.05	0.17
VAYZ	1,783,010	0.15	0.11	2.07	0.06	0.20
VEGFF	1,783,010	0.10	0.07	2.59	0.03	0.13
VEARS	1,783,010	0.12	0.09	1.53	0.05	0.17
VEAYZ	1,783,010	0.15	0.12	1.62	0.06	0.20
RET	1,782,976	0.01	0.01	5.54	-0.06	0.07
ME	1,429,915	5.56	5.42	0.27	4.08	6.87
BM	1,203,196	-0.50	-0.54	0.94	-1.11	-0.02
TURN	1,563,833	2.11	2.17	-0.11	1.37	2.90
CVTURN	1,563,833	4.07	4.06	0.35	3.70	4.42
CRET	1,518,318	1.06	1.03	1.22	0.86	1.19
BETA	1,564,562	1.18	1.13	0.23	0.81	1.51

Table 3
Sample Correlations

Variable	VAFF	VARS	VAYZ	VEGFF	VEARS	VEAYZ	RET
VAFF	1.00	0.77*	0.81*	0.28*	0.61*	0.62*	-0.07*
VARS		1.00	0.93*	0.31*	0.81*	0.79*	-0.05*
VAYZ			1.00	0.29*	0.75*	0.76*	-0.05*
VEGFF				1.00	0.31*	0.31*	0.00
VEARS					1.00	0.94*	-0.04*
VEAYZ						1.00	-0.04*
RET							1.00

***Significant at the 1% level**

Table 4
Regression Results

The dependent variable is the return in the month for which expected volatilities are computed by employing the EGARCH(p,q) model (*VEGFF*) and the ARIMA(p,q) model (*VEARS* and *VEAYZ*). The EGARCH model uses the residuals from the monthly regressions of monthly stock returns on the three Fama-French factors. The ARIMA models use the realized volatilities computed using the Rogers and Satchell (1991) approach (*VEARS*) and the Yang and Zhang (2000) approach (*VEAYZ*). *BETA*, *BM*, and *ME* are calculated by following the Fama-French (1992) method. *CRET* is the cumulative gross returns in the six-month period T-7 to T-2. *TURN* is the natural log of the average percentage turnover ratio, defined as (shares traded/number of outstanding shares), for a stock in the previous 36 months and *CVTURN* is the natural log of coefficient of variation of the turnover ratios in the previous 36 months. Numbers in parentheses are the t-statistics computed using the Fama and Macbeth (1973) method.

BETA	BM	ME	CRET	TURN	CVTURN	VAFF	VARS	VAYZ	VEGFF	VEARS	VEAYZ	R^2 (%)
0.005	0.001	0.000										3.5
(1.23)	(1.88)	(-0.21)										
0.003	0.000	0.000	**0.155**	0.001	**0.004**							17.7
(1.07)	(-0.34)	(0.03)	(62.84)	(1.04)	(2.73)							
0.004	**-0.001**	**-0.002**	**0.153**	**0.002**	**0.006**	**-0.068**						18.9
(1.61)	(-2.04)	(-4.24)	(67.04)	(2.11)	(4.27)	(-6.27)						
0.003	-0.001	-0.001	**0.153**	**0.001**	**0.004**		-0.014					19.0
(1.20)	(-1.36)	(-1.48)	(67.12)	(2.14)	(3.20)		(-1.02)					
0.003	-0.001	**-0.001**	**0.153**	0.001	**0.004**			**-0.018**				18.8
(1.28)	(-1.31)	(-2.11)	(66.93)	(1.83)	(3.47)			(-1.94)				
0.002	0.000	0.000	**0.155**	0.001	**0.004**				**0.013**			17.9
(0.90)	(-0.11)	(0.33)	(63.43)	(0.82)	(2.62)				(3.77)			
0.003	0.000	0.000	**0.155**	0.001	**0.004**					0.002		18.7
(1.03)	(-0.22)	(-0.13)	(65.90)	(1.14)	(3.02)					(0.12)		
0.003	0.000	0.000	**0.155**	0.001	**0.004**						0.000	18.6
(1.06)	(-0.27)	(-0.25)	(65.74)	(1.08)	(3.09)						(0.00)	

Numbers in bold are significant at better than 5% level